

O. Scavenius^a, A. Dumitru^b, A.D. Jackson^c^a *NORDITA, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark*^b *Department of Physics, Columbia University, 538 W. 120th Street, New York, NY 10027, USA*^c *The Niels Bohr Institute, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark*

Recent results from Au+Au collisions at BNL-RHIC energy hint at explosive hadron production at the QCD transition rather than soft hydrodynamic evolution. We speculate that this is due to a rapid variation of the effective potential for QCD close to T_c . Performing real-time lattice simulations of an effective theory we show that the fast evolution of the potential leads to “explosive” spinodal decomposition rather than bubble nucleation or critical slowing down.

It is hoped that heavy ion collision experiments at BNL-RHIC and later at CERN-LHC will provide evidence for the deconfined phase of QCD [1]. Lattice gauge simulations predict a weakly first-order deconfinement transition at a critical temperature of $T_c \sim 0.63\sqrt{\sigma}$, where $\sigma \sim 1$ GeV/fm is the string tension at $T = 0$. For QCD with quarks, the transition could remain weakly first-order or become a smooth crossover. Even so, lattice data for two or three light flavors [2] indicate a steep rise in energy density and pressure with temperature and thus a rapid shift of the global minimum of the effective thermodynamic potential for QCD within a narrow temperature interval centered at T_c . To date, it has been difficult to identify unambiguous observables providing compelling evidence for a phase transition in ultrarelativistic heavy ion collisions (URHIC). However, rapid changes in the QCD effective potential near T_c and/or supercooling followed by spinodal decomposition can have distinctive signatures. Their experimental observation would probably represent the most direct information which URHIC events can provide regarding the QCD phase transition. The purpose of this Letter is to consider the likelihood of explosive “spinodal” decomposition in RHIC experiments.

In a first-order phase transition, the thermodynamic potential for T slightly less than T_c exhibits a metastable minimum corresponding to the deconfined phase. At lower temperatures, this minimum becomes a point of inflection, i.e. a “spinodal instability”. In a slowly expanding system, such as the early universe, the phase transition proceeds through the nucleation of bubbles of the “true vacuum” state via thermal activation for all $T < T_c$ [3]. In ultrarelativistic collisions of heavy ions or hadrons, expansion is very fast and the metastable deconfined state may reach the inflection point and undergo spinodal decomposition before significant nucle-

ation has taken place. Calculations using an effective chiral model [4] suggest that spinodal decomposition is the favored mechanism in URHIC. The condensate field in each causally connected region “rolls down” to its minimum with large attendant fluctuations in the chiral order parameter and related hadronic observables.

Effects similar to those of spinodal decomposition can also be expected, independent of the order of the phase transition, for any rapidly varying thermodynamic potential. In this case, fields are unable to follow the rapid evolution of the equilibrium potential, and the system decays into any of a large number of unstable pion and sigma modes. (By contrast, the true spinodal decomposition of a slowly varying potential proceeds via the “critical” growth of long-wavelength modes of the order parameter [5]. Here, we shall use the term “spinodal” somewhat loosely to describe any rapid decomposition.) This was first suggested in [6] in the context of chiral symmetry breaking. It was argued that, if expansion is sufficiently fast, the system can cool significantly while the chiral order parameter remains near its restored symmetry value. Violent decomposition would follow as a natural consequence of such a “quench”. Subsequent numerical simulations [7] showed that, if chiral symmetry breaking is considered independent of confinement, the chiral effective potential changes too slowly with temperature for such effects to be seen in URHIC. If, however, the effective potential changes rapidly with temperature [8], the quenched scenario and effective spinodal decomposition could be an interesting possibility even for moderate expansion rates.

RHIC ($\sqrt{s} = 130$ A GeV) has already produced a wealth of new results [9]. “Dynamical” fluctuations in the average transverse momentum, p_t , are larger than at the SPS ($\sqrt{s} = 17$ A GeV). Observed Hanbury-Brown-Twiss (HBT) radii from pion interferometry are small, $\sim 5 - 7$ fm, and average transverse boost velocities are large, $\sim 0.6c$. In particular, the fact that the HBT outward and sideward radii, R_o and R_s , are roughly equal for pion pair transverse momenta in the range $100 < p_t < 500$ MeV may indicate explosive decomposition. These phenomena all hint at a rapid out-of-equilibrium transition. By contrast, if a first-order transition proceeds at equilibrium, a mixed phase is created, and the system should spend a long time at [10] and slightly below [11] T_c . As a result, the outward HBT radius R_o should be larger than the sideward radius R_s by as much as 50% – 100% [10,11].

We wish to consider the possibility that the evolution of central RHIC events is determined by rapid changes in the thermodynamic potential. This would be the case, for example, if the chiral phase transition were driven by the confinement transition. The familiar rapid change of the energy density with temperature would then be reflected in equally rapid changes in the thermodynamic potential for the chiral fields. To illustrate this point, we will perform real time simulations of a weakly first-order deconfinement transition using a recently suggested effective theory, valid near T_c , in which the free energy (or pressure) is dominated by a condensate of Polyakov loops ℓ [12]. The potential for ℓ is taken as [12]

$$\mathcal{V}(\ell) = \left(-\frac{b_2}{2}|\ell|^2 - \frac{b_3}{6}(\ell^3 + (\ell^*)^3) + \frac{1}{4}(|\ell|^2)^2 \right) b_4 T^4, \quad (1)$$

which is invariant under global $Z(3)$ transformations. The form of $\mathcal{V}(\ell)$ is dictated by usual symmetry principles [12]. The center symmetry of $SU(3)$, $Z(3)$, is broken at high T where the global minimum of $\mathcal{V}(\ell)$, $\ell_0(T) \rightarrow 1$, and restored at low T where $\ell_0(T) \rightarrow 0$.

The assumption that quarks are not important for understanding the form of the effective potential for QCD is motivated by the results of lattice simulations. Specifically, plots of P/P_{ideal} versus T/T_c are remarkably insensitive to the number of flavors, $N_f = 0, 2, 3$ [2]. Thus, we neglect terms linear in ℓ , which must exist in QCD with quarks. Such terms would change the first-order transition (for $N_f = 0$) into a crossover. Nevertheless, the fact that the free energy is small below T_c means that their numerical effects must be small. We emphasize that the results (for rapid expansion) to be described below are *not* driven by the order of the phase transition and would not be changed in any significant way by the inclusion of such linear terms.

The coefficients b_2 , b_3 , and b_4 can be chosen to reproduce lattice data for the pressure and energy density of pure glue theory for $T \geq T_c$ [8]. In practice, to reproduce the pressure and energy density we may choose constant $b_3 \approx 0.9$ and $b_4 \approx 15.0$. For $N_f = 3$, we rescale b_4 by a factor $47.5/16$, corresponding to the increase in the number of degrees of freedom. In the spirit of mean field theory, the same constant values of b_3 and b_4 are to be employed for $T < T_c$. In this domain, $b_2(T) \approx -0.66\sigma^2(T)/T^4$ can be set using the finite- T string tension $\sigma(T)$ from the lattice [2].

The potential in eq. (1) changes extremely rapidly near T_c , see Fig. 1 of [8]. For temperatures even $\pm 2\%$ away from T_c , the potential has only a single minimum, which corresponds to the deconfined (confined) state. This rapid change in the effective potential with temperature, which is directly related to the rapid increase of $\sigma(T)$ below T_c seen in lattice data [2], makes “explosive” decomposition inevitable. (The rapid variation of $b_2(T)$ about T_c can also be understood qualitatively by considering the above mean field theory for the second-order

confinement transition, $b_3 = 0$, for the quenched theory with two colors. Then, $b_2(T) = c(T/T_c - 1)$. The rapid rise of the pressure relative to that for an ideal gas about T_c [13], $P(T)/P_{\text{id}}(T) = b_2^2(T)$, implies that c is large.)

To complete the effective theory, we add a kinetic term for ℓ and allow coupling to a chiral field, ϕ , describing the usual pions and the sigma meson [8]. Thus,

$$\mathcal{L} = \mathcal{L}_\phi + \frac{N_c}{g^2} |\partial_\mu \ell|^2 T^2 - \mathcal{V}(\ell) - \frac{h^2}{2} \phi^2 |\ell|^2 T^2. \quad (2)$$

\mathcal{L}_ϕ is the standard Lagrangian for the chiral fields, see e.g. [4,6,7]. The coefficient of the kinetic term for ℓ is set here by the fluctuations of $SU(3)$ Wilson lines in space. Perturbative corrections to that coefficient are small [14]. Thus, we assume a Lorentz invariant structure [8]. The coupling $h^2 \simeq 22$ between the chiral field and ℓ is chosen to reproduce $m_\pi(T)$ [15]. This interaction term allows the Polyakov loop condensate to drive the chiral phase transition. While the inclusion of the chiral fields enables us to calculate familiar observables, it is otherwise largely passive. The behavior of ℓ , i.e. confinement physics or its $Z(3)$ symmetry properties control the QCD transition. (In this regard, see also [16].)

In our numerical simulations we solve the classical Euler-Lagrange equations derived from the Lagrangian (2) on a 64^3 space-like lattice. We employ a Robertson-Walker metric $ds^2 = d\tau^2 - a^2(\tau)d(\tau_0\eta)^2 - dx_\perp^2$ with scale factor $a(\tau) = \tau/\tau_0$ and a Hubble constant $H = d\log(a)/d\tau = 1/\tau$. Here, τ and η are the longitudinal proper time and the space-time rapidity, respectively [17]. We can vary the expansion rate by varying the initial time, τ_0 . Further, we impose periodic boundary conditions and choose a lattice spacing $a_L = 0.25$ fm. We draw random initial field values at each lattice site from a Gaussian distribution and then coarse grain the lattice afterwards to a correlation length of $2a_L$. The initial temperature is chosen to be $T(\tau_0) = 1.01 T_c$. For the chiral field fluctuations, we assume [6] that $\langle \phi \rangle = \langle \dot{\phi} \rangle = 0$, $\langle \phi^2 \rangle = v^2/4$, and $\langle \dot{\phi}^2 \rangle = v^2/\text{fm}^2$ (v is the vacuum expectation value of the sigma field, $\sigma \equiv \phi_0$). The ℓ -field is initialized in the deconfined minimum of the potential with real phase, $\langle \ell \rangle \approx 0.7$. The magnitude of the initial fluctuations, $\langle \ell^2 \rangle - \langle \ell \rangle^2 \approx 0.1$ was chosen such that the initial energy density is $\approx 4T_c^4$ [2]. The results are essentially independent of the initial distribution of energy between fluctuations of $\ell(x)$ and $\dot{\ell}(x)$.

The presence of fluctuations modifies the potential in eq. (1). We renormalize it by expanding $\mathcal{V}(\ell, \phi)$ at $\tau = \tau_0$ to second order in the fluctuations $\delta\ell$ and $\delta\phi$ and then subtract the resulting $\delta\mathcal{V}(\ell, \phi; \delta\ell, \delta\phi)$. This provides us with the renormalized equations of motion.

We show the results of simulations for two different initial times; one appropriate for “slow” expansion ($\tau_0 = 2000$ fm/c) and another ($\tau_0 = 100$ fm/c) for “fast” expansion. These two cases will suffice to illustrate the

radical changes which occur in the nature of the transition as a function of the expansion rate. The slow expansion is, of course, many orders of magnitude faster than that describing the QCD transition in the early universe, while the fast expansion is slow on typical URHIC scales, and thus the indicators of non-equilibrium phenomena shown below should be even stronger in real URHIC.

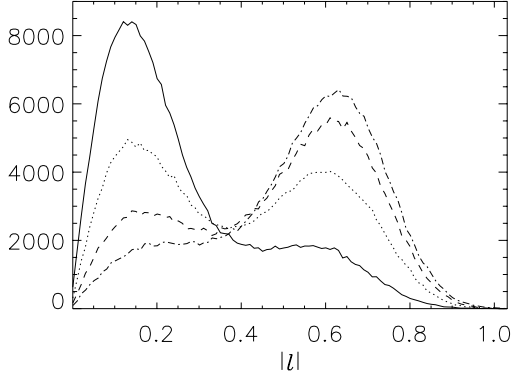


FIG. 1. $|\ell|$ field histograms obtained for the slow expansion. The times are $a(\tau) \equiv \tau/\tau_0 = 1.0443, 1.0456, 1.0467$ and 1.0480 , from bottom to top in the left peak, respectively.

The potential energy of the ℓ -field in the co-moving frame is assumed to red-shift according to $T^4/T_0^4 = 1/a(\tau) \equiv \tau_0/\tau$. However, the spectrum of long wavelength fluctuations on the lattice is determined by the dynamics of the phase transition; for example, in case of slow expansion with nucleation and bubble growth those fluctuations on the lattice may not cool during the phase conversion. Only hard, short wavelength fluctuations which are integrated out and determine the coefficients in the effective potential (1), but are not treated explicitly in the classical lattice simulation, cool according to the above prescribed $T(\tau)$ law.

Fig. 1 shows a sequence of histograms describing the distribution of $|\ell|$ for slow expansion. The transition is weakly first order and, accordingly, the effective potential for ℓ exhibits two local minima for T in the immediate vicinity of T_c . Although $\langle \ell(x) \rangle$ is initially localized in the deconfined minimum at $\ell_0 \sim 0.7$, the fluctuations serve as a heat bath and trigger thermal activation over the barrier. Fig. 1 shows that the confined and deconfined states coexist in different parts of the system. Well-defined bubbles form and, as time progresses, grow, coalesce, and eventually complete the transition to confinement.

Fig. 2 shows the corresponding histograms of $|\ell|$ for the fast expansion. Clearly, the double peak has disappeared, and the distributions merely broaden and move towards the confined minimum as the system cools. In this case, we conclude that the phase conversion happens through rapid decomposition. For a rough estimate of the relevant time scales, note that during a small time interval $\Delta\tau$, the hard modes cool by $|\Delta T|/T_c \simeq \Delta\tau/4\tau_0$. From classical nucleation theory [3,4] a lower bound on the time scale

for nucleation is $\Gamma^{-1/4} \gtrsim 1/T_c$. For the model (1) with $b_2(T)$ set by the string tension $\sigma(T)$ [2], supercooling by $\sim 1\%$ suffices for the soft modes to “roll down” to the confined minimum [8] rather than nucleation to occur. Setting $\Delta\tau T_c \simeq 1$, we expect that for $\tau_0 \lesssim 100$ fm/c phase coexistence is not established.

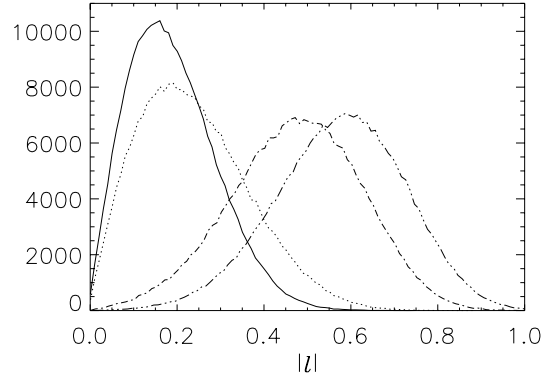


FIG. 2. $|\ell|$ field histograms obtained from the “fast” lattice simulation. The times are $a(\tau) \equiv \tau/\tau_0 = 1.0493, 1.0616, 1.0739$ and 1.0861 from right to left, respectively.

Fig. 3 describes the time evolution of $\langle |\ell(x)| \rangle$, and its root mean square (RMS) fluctuations. In the slow case, we see that the mean field makes the transition from the deconfined minimum to the confined minimum very smoothly. The corresponding RMS fluctuations increase immediately before phase conversion and illustrate the competition between the two minima. (For a second-order phase transition and very slow expansion those fluctuations would go “critical” [5]). After the transition, the mean field shows no temporal oscillations and only small spatial fluctuations, indicating that the system is quite homogeneous after the transition.

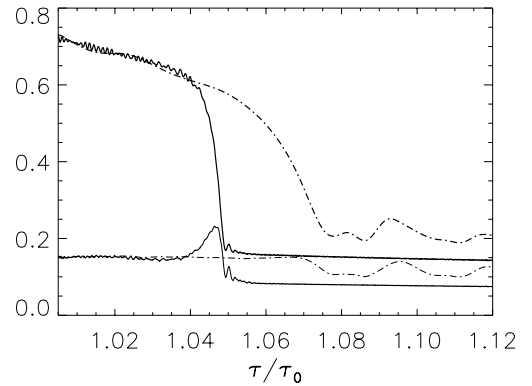


FIG. 3. The mean field, $\langle |\ell(x)| \rangle$, (thick lines) and its RMS fluctuations in space (thin lines). The solid (dashed-dotted) curves describe slow (fast) expansion.

By contrast, fast expansion leads to strong temporal oscillations of the mean field and large spatial fluctuations following the confinement transition. Thus, the scale of spatial homogeneity is smaller in this case. Also

note that the rapid phase conversion and decoherence seen for fast expansion lead to the violent and rapid decay of the Polyakov loop condensate into physical particles. This, too, suggests explosive decomposition of the source. As discussed previously, strong event-by-event fluctuations of the pion mean- p_t should emerge as a consequence [8]. Also, the “sudden” transition could help to understand the flavor composition in the final state [19].

Fig. 4 summarizes the corresponding behavior of the chiral σ field. The qualitative pattern here is the same, but the effects are enhanced. This indicates that substantial energy has been flushed quickly from the Polyakov loop condensate into the chiral field. This helps justify our expectation of the approximate equality of outward and sideward HBT radii: The outward radius R_o^2 is larger than the sideward radius R_s^2 by an amount roughly equal to the variance of the pion production time $\delta\tau^2 = \langle\tau^2\rangle - \langle\tau\rangle^2$ [10,11,18]. For a smooth quasi-adiabatic transition, entropy conservation in a comoving volume element requires $\delta\tau^2$ to be large, while the non-equilibrium transition seen in our simulations is very fast.

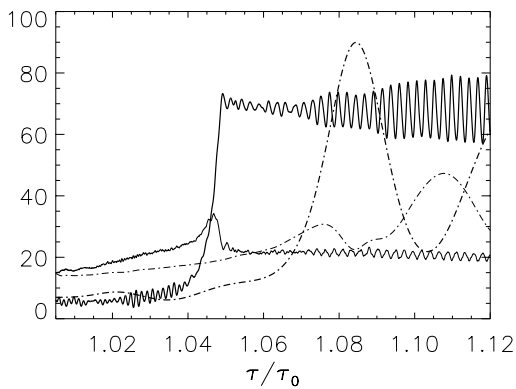


FIG. 4. The mean field $\langle\sigma(x)\rangle$ (thick lines) and RMS fluctuations (thin lines) in MeV. The solid curves describe slow expansion. The dashed curves describe fast expansion.

To summarize, real-time simulations of the Polyakov loop condensate model have demonstrated that radically different phase transition dynamics can emerge in URHIC for realistically fast expansion rates when the effective potential changes rapidly in the vicinity of T_c . The model considered is consistent with lattice QCD data for the pressure, energy density, and correlation length (or string tension) near T_c . In real time, the picture that emerges is a confinement transition via spinodal decomposition which then triggers chiral symmetry breaking. The timescales involved in URHIC seem much too short to admit the traditional near equilibrium transition via bubble nucleation. As expected in any case of phase conversion via spinodal decomposition, we observe large RMS spatial fluctuations in the chiral fields after the transition, i.e. a small spatial homogeneity length. The near equivalence of outward and sideward HBT radii then follows as a natural consequence of the almost in-

stantaneous decay of the Polyakov loop condensate into an “exploding” source of pions [18]. Large event-by-event fluctuations of $\langle p_t \rangle$ are another outcome of such a strong non-equilibrium transition.

ACKNOWLEDGMENTS

We thank J. Borg, D. Diakonov, H. Heiselberg, L. McLerran, I.N. Mishustin, R.D. Pisarski, D. Son, and R. Venugopalan for helpful criticism and discussions. A.D. gratefully acknowledges the support of the US-DOE through Contract No. DE-FG-02-93ER-40764.

-
- [1] J. Harris and B. Müller, *Ann. Rev. Nucl. Part. Sci.* **46**, 71 (1996).
 - [2] O. Kaczmarek, F. Karsch, E. Laermann and M. Lutgemeier, *Phys. Rev. D* **62**, 034021 (2000); F. Karsch, E. Laermann and A. Peikert, *Phys. Lett. B* **478**, 447 (2000).
 - [3] K. Kajantie and H. Kurki-Suonio, *Phys. Rev. D* **34**, 1719 (1986).
 - [4] O. Scavenius, A. Dumitru, E. S. Fraga, J. T. Lenaghan and A. D. Jackson, *hep-ph/0009171*.
 - [5] B. Berdnikov and K. Rajagopal, *Phys. Rev. D* **61**, 105017 (2000).
 - [6] K. Rajagopal and F. Wilczek, *Nucl. Phys.* **B404**, 577 (1993).
 - [7] J. Randrup, *Phys. Rev. Lett.* **77**, 1226 (1996); I. N. Mishustin and O. Scavenius, *Phys. Lett. B* **396**, 33 (1997).
 - [8] A. Dumitru and R. D. Pisarski, *hep-ph/0010083*.
 - [9] J. Harris, F. Laue, J. Reid, S. Panitkin, and N. Xu, Talks given at “Quark Matter 2001”, Jan. 14th – 20th, 2001, Stony Brook, New York; <http://www.rhic.bnl.gov/qm2001>.
 - [10] S. Pratt, *Phys. Rev. D* **33**, 1314 (1986); C. M. Hung and E. V. Shuryak, *Phys. Rev. Lett.* **75**, 4003 (1995); D. H. Rischke and M. Gyulassy, *Nucl. Phys. A* **597**, 701 (1996); *Nucl. Phys. A* **608**, 479 (1996).
 - [11] S. Soff, S.A. Bass, and A. Dumitru, *nucl-th/0012085*.
 - [12] R. D. Pisarski, *Phys. Rev. D* **62**, 111501 (2000).
 - [13] J. Engels, J. Fingberg, K. Redlich, H. Satz and M. Weber, *Z. Phys. C* **42**, 341 (1989).
 - [14] J. Wirstam, *hep-ph/0106141*.
 - [15] R. V. Gavai and S. Gupta, *Phys. Rev. Lett.* **85**, 2068 (2000).
 - [16] I. N. Mishustin and O. Scavenius, *Phys. Rev. Lett.* **83**, 3134 (1999).
 - [17] J. D. Bjorken, *Phys. Rev. D* **27**, 140 (1983).
 - [18] H. Heiselberg and A. D. Jackson, *nucl-th/9809013*.
 - [19] T. Csörgő and L. P. Csernai, *Phys. Lett. B* **333**, 494 (1994); R. Stock, *Phys. Lett. B* **456**, 277 (1999); J. Rafelski and J. Letessier, *Phys. Rev. Lett.* **85**, 4695 (2000).

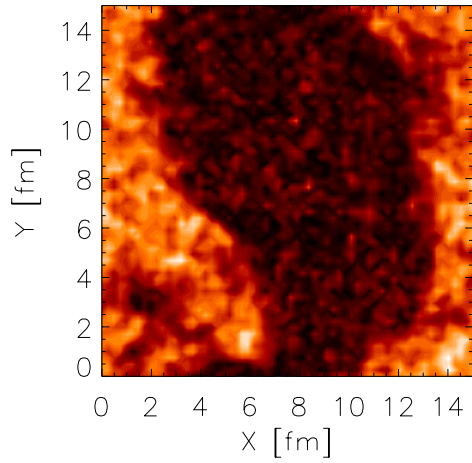
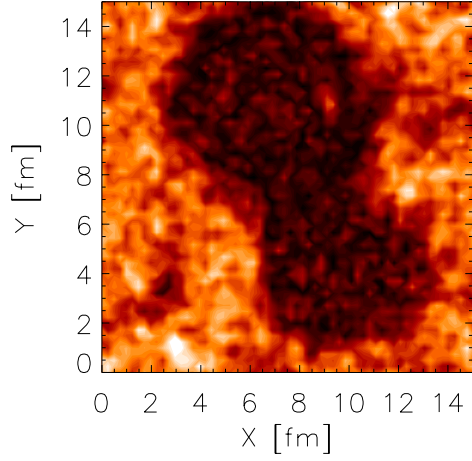
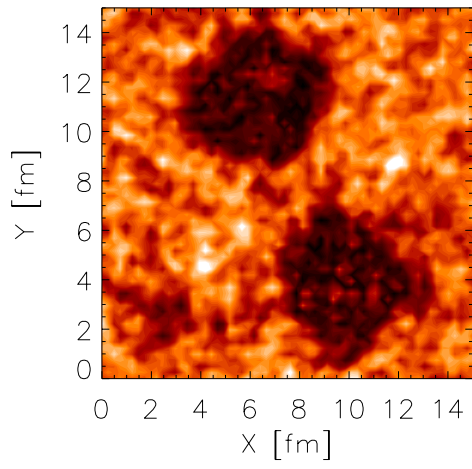


FIG. 5. Additional figure: $|\ell|$ contours in the transverse plane, $\eta = 0$, for the slow expansion, $\tau_0 = 1970$ fm. The times are $a(\tau) \equiv \tau/\tau_0 = 1.045, 1.04875, 1.05031$, from top to bottom. Dark color corresponds to small ℓ (confined phase) while light color corresponds to large ℓ (deconfined phase).

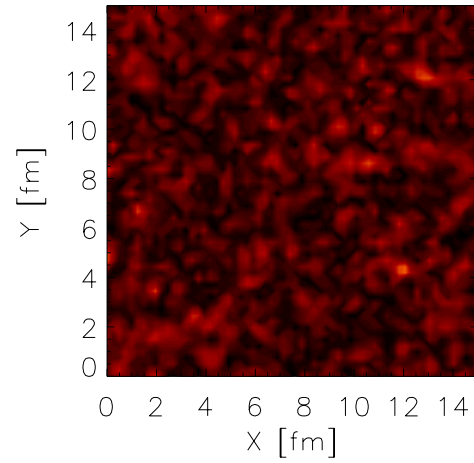
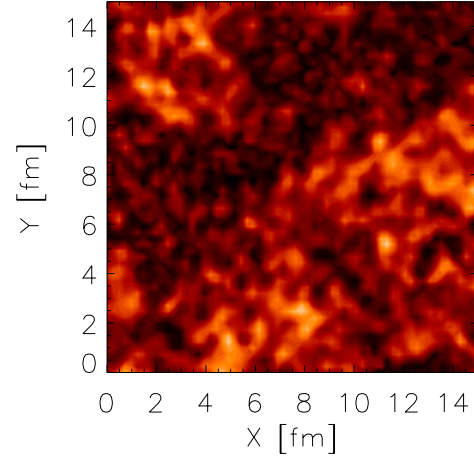
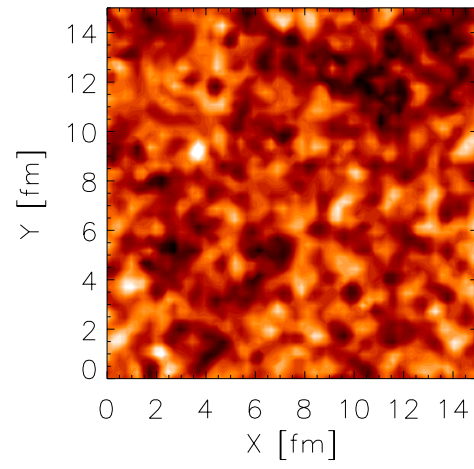


FIG. 6. Additional figure: $|\ell|$ contours in the transverse plane, $\eta = 0$, for the fast expansion, $\tau_0 = 98$ fm. The times are $a(\tau) \equiv \tau/\tau_0 = 1.056, 1.069, 1.082$, from top to bottom. Dark color corresponds to small ℓ (confined phase) while light color corresponds to large ℓ (deconfined phase).